Math 3150 Exam 3C, April 2026, 12 problems, 75 minutes

Name: Answer Key

Read these instructions:

- Leaving the testing room results in a new exam given for the unfinished problems.
- Two detached sheets of notes allowed.
- No electronics.
- Raise your hand for questions or more paper.

Problem 1. Write the following statements in terms of the letters $a =$ "Ann is a math major", $b =$ "Ben is a math major", $d =$ "Dan is a math major" and the symbols $\neg$, $\wedge$, $\vee$, $\oplus$.

+2 **Part A.** "Either Dan or Ben is a math major." $d \vee b$ OR $d \oplus b$

+2 **Part B.** "Neither Dan nor Ann are math majors." $\neg(d \vee a)$ OR $\neg d \wedge \neg a$

+2 **Part C.** "Ben is not a math major." $\neg b$

+2 **Problem 2A.** State the **converse** of "If there is smoke, then there is fire" in English.

If there's fire, then there's smoke

+2 **Problem 2B.** State the **contrapositive** of "If there is smoke, then there is fire" in English.

If there's no fire, then there's no smoke

+3 **Problem 3A.** Write the truth table for $(p \Rightarrow q) \wedge (q \Rightarrow p)$.

p	q	$p \Rightarrow q$	$q \Rightarrow p$	$(p \Rightarrow q) \wedge (q \Rightarrow p)$
T	T	T	T	T
T	F	F	T	F
F	T	T	F	F
F	F	T	T	T

+2 **Problem 3B.** Find a disjunctive normal form for $(p \Rightarrow q) \wedge (q \Rightarrow p)$.

$(p \wedge q) \vee (\neg p \wedge \neg q)$

+5 **Problem 4.** Is $(p \Rightarrow q) \wedge \neg p$ logically equivalent to $\neg p$? Explain.

p	q	$p \Rightarrow q$	$\neg p$	$(p \Rightarrow q) \wedge \neg p$
T	T	T	F	F
T	F	F	F	F
F	T	T	T	T
F	F	T	T	T

Yes because final two columns are identical

+2 **Problem 5.** Let $T(x)=$"x has thorns", $P(x)=$"x has petals", and R be the set of all roses. Write "If every rose has thorns, then every rose has petals" in terms of x, $T(x)$, $P(x)$, R, $\forall$, $\exists$, $\in$, :, [,], $\wedge$, $\vee$, $\neg$, $\Rightarrow$.

$[\forall x \in R : T(x)] \Rightarrow [\forall x \in R : P(x)]$

Problem 6. Let $B(x) =$ "x has at least one bird", $C(x) =$ "x has at least one cat", $D(x) =$ "x has at least one dog", and S be the set of students in this class. Formalize (a)-(c) via the symbols $S, x, B(x), C(x), D(x), \wedge, \vee, \neg, \Rightarrow, \forall, \exists, :$ (colon), $\in$, [,].

+2 **Part A.** Every student in this class has at least one bird and no cat.

$\forall x \in S : [B(x) \wedge \neg C(x)]$

+2 **Part B.** Some student in this class has at least one bird, at least one cat, or at least one dog.

$\exists x \in S : [B(x) \vee C(x) \vee D(x)]$

+2 **Part C.** Every student in this class has at most one type of the animals out of birds, cats, and dogs.

$\forall x \in S : [B(x) \wedge \neg C(x) \wedge \neg D(x)] \vee [\neg B(x) \wedge C(x) \wedge \neg D(x)] \vee [\neg B(x) \wedge \neg C(x) \wedge D(x)] \vee [\neg B(x) \wedge \neg C(x) \wedge \neg D(x)]$

+2 **Problem 7.** Which option below is equivalent to the **negation** of "Some teapot is either short or stout"?

every notshort and not stout

(a) Some teapot is not short and not stout.
(b) Some teapot is not short or not stout.
(c) Every teapot is not short and not stout.
(d) Every teapot is not short or not stout.

Problem 8. Mark each item as true or false. Justify your answers.

+2 **Part A.** $\exists x \in \mathbb{Q} : \frac{1}{x} > x$ True : let $x = \frac{1}{2}$. Then $\frac{1}{x} = 2 > \frac{1}{2}$.

+2 **Part B.** $\exists x \in \mathbb{R} : [\forall y \in \mathbb{R} : x^2 + y = 0]$ False : if $x^2 + y = 0$ then $x^2 + (y+1) = 1 \neq 0$, so if an x works for one y it doesn't work for a different value of y.

+2 **Part C.** $\forall y \in \mathbb{R} : [\exists x \in \mathbb{R} : x - y \geq 0]$

~~False~~ True : pick $x = y$. Then $x - y = y - y = 0 \geq 0$ ✓.

+2 **Part D.** $\exists y \in \mathbb{R} : [\forall x \in \mathbb{R} : x - y \geq 0]$ False : given such a y, pick $x = y - 1$. Then $x - y = (y-1) - y = -1 \not\geq 0$.

+2 **Part E.** $\exists x \in \mathbb{R} : [(x^2 = -1) \Rightarrow (x > 0)]$ True : let $x = 0$. Then $(x^2 = -1) = F$, $(x > 0) = F$, so $[(x^2 = -1) \Rightarrow (x > 0)] = [F \Rightarrow F] = $ True.

Problem 9. Is the function $f : \{a, b, c, d\} \to \{u, v, w\}$ defined by $f(a) = u$, $f(b) = v$, $f(c) = w$, $f(d) = u$ onto? Explain.

+2 Yes: every codomain element u, v, w is an output.

+4 **Problem 10A.** Is the function $g : \mathbb{R} \to \mathbb{R}$ defined by $g(x) = 2 \cdot (1 - x)$ one-to-one? Justify your answer.

Yes: $g(x_1) = g(x_2) \Rightarrow 2 \cdot (1 - x_1) = 2 \cdot (1 - x_2) \overset{\div 2}{\Rightarrow} 1 - x_1 = 1 - x_2 \overset{-1}{\Rightarrow} -x_1 = -x_2 \overset{\cdot -1}{\Rightarrow} x_1 = x_2$. So g is one-to-one.

+4 **Problem 10B.** Is the function $h : \mathbb{Z} \to \mathbb{Z}$ defined by $h(x) = 2 \cdot (1 - x)$ onto? Justify your answer.

No: we show 1 is not an output: $1 = h(x) \Rightarrow 1 = 2 \cdot (1 - x) \overset{\div 2}{\Rightarrow} \frac{1}{2} = 1 - x \Rightarrow x = \frac{1}{2} \notin \mathbb{Z} = \text{domain}(h)$.

So h is not onto.

+2 **Problem 11.** Suppose that p and q are statements such that $p \Rightarrow q$ is true.

From truth table of $p \Rightarrow q$, this happens if ① $p = T, q = T$ or ② $p = F, q = T$ ~~or F~~, ③ $p = F, q = F$

Part A. Find the truth value of $p \vee q$. Justify your answer.

Case 1: $p = T, q = T$. Then $p \vee q = T \vee T = T$

Case 2: $p = F, q = T$ $\quad p \vee q = T$

Case 3: $p = F, q = F$ $\quad p \vee q = F$

So $p \vee q = T$ or F.

+2 **Part B.** Find the truth value of $p \wedge \neg q$. Justify your answer.

Case 1: $T \wedge \neg T = T \wedge F = F$

Case 2: $F \wedge \neg T = F \wedge F = F$

Case 3: $F \wedge \neg F = F \wedge T = F$

So $p \wedge \neg q = F$.

Problem 12. Consider the function $f : \mathbb{Z} \to \mathbb{Z}$ defined by $f(n) = n^2$.

+3 **Part A.** Is the statement $\underbrace{\forall x \in \mathbb{Z} : [\forall y \in \mathbb{Z} : (f(x) = f(y)) \Rightarrow (x = y)]}_{\text{"f is one-to-one"}}$ true or false? Justify your answer.

False: $f(1) = 1^2 = f(-1)$ but $1 \neq -1$.

+3 **Part B.** Is the statement $\underbrace{\forall y \in \mathbb{Z} : [\exists x \in \mathbb{Z} : y = f(x)]}_{\text{"f is onto"}}$ true or false? Justify your answer.

False: -1 is not an output of f since outputs of $f(n) = n^2 \geq 0$.

Score	20	20	20